Calculus BC Syllabus 2019-2020

Course Design and Philosophy

Calculus is a course that consists of the application of three main ideas: limits, slopes and sums. Each of these main topics includes its own universe of applications, spread across all possible mathematical functions. My goal is to teach students the reasons behind the applications, in addition to the specific applications themselves. Once students begin to grasp the reasons, the applications follow nicely, assuming a rigorous previous mathematical background. Applications to physical and scientific models are discussed, as well.

Teaching Strategies

Graphing calculator

Students are required to have a TI-type graphing calculator. The calculator is a tool, a means to an end. Direct instruction on the use of this tool is required at certain points: to set windows for graphing, to use NDER and NINT, and to make sure that all modes are appropriately set. The calculator is used for numerical analysis and graphical presentation of functions. Examples of NDER failing because of non-differentiable functions will also be presented, so that students are aware of the methods that the calculator itself uses to evaluate information. The calculator is an excellent tool for displaying graphs of functions. Students will be required to use the calculator to recognize asymptotes, critical points, and points of inflection, as well as discontinuities, cusps, and similar non-differentiable points. Graphs will be used to evaluate the reasonableness of a solution and to estimate locations of critical points and roots. Graphs, generated by the graphing utility www.desmos.com and by the graphing utility on the TI-series graphing calculator will be used to experiment and observe functions. As an example, early in the year, after the study of limits, students will be asked to graph a function and its tangent line at a point. They will then zoom in to this point, observing the local linearity of the function at the (differentiable) point and the approach of this linear behavior to the previously identified tangent line. As another example, in conjunction with learning about inflection points, students will be given an abstract cubic $Ax^3 + Bx^2 + Cx + D$. They will be asked to experiment by variying the parameters in order to find the conditions under which extrema exist, using their calculator to graph the cubic. A deeper question will be asked about the inflection point (which always exists, for non-zero A. Even if extrema don't exist, the inflection will exist with a cubic -- a subtle but important point.)

A calculator serves as a tool that can be used to get to know "families" of functions. Students should know the shape and behavior of exponentials/logs, logistics, trignometric functions, fractional-exponent polynomials, and whole-number-exponent polynomials, at the very least.
Four different ways of knowing There are four methods of analysis that we use in class: numerical (data tables, iterative methods), analytical (function-based), graphical, and verbal.

1) Graphs are used to establish a visual picture of a mathematical idea or function. Students are asked to estimate roots, critical points, and inflection points given a graph. Students are

also asked to generate derivative graphs from a function graph, and, later in the course, to find the graph of the anti-derivative. Applications: MVT, slope fields, definite integrals, derivatives.
2) Iterative methods are used when functions exhibit curious behavior around certain points, or there is data that is collected with actual "real-world" sensors or measurements. Tables of collected values are used to generate an estimated derivative graph, and are also used to generate tables of definite integrals. Applications: MVT, average velocity, data tables collected using Vernier probes.
3) Analytical methods are what students often associate with "real" math. The algorithmic methods used in evaluating a function analytically are often mistaken for the heart of mathematics. To guard against this, different variables are used (for example, s(t) instead of y(x)). The applications are myriad: computing derivatives and integrals (see topical list), using the product rule, the quotient rule, the chain rule, indefinite integrals.

4) Verbal explanations establish the presence of conceptual understanding behind the solution to a problem. Applications in this course include explaining the units that might be used to describe a numerical solution, explaining the meaning of a derivative, and being able to articulate the meaning of velocity, acceleration, and rate information.

AP exam-specific information

When students sit for the AP exam, they will be asked to provide all four types of solutions to problems. From chapter 4 onward, all tests will include at least one question directly from or directly inspired by a past AP problem. Throughout the year, information about the administration of the AP exam, topics on the AP exam, and problems from the AP exam will be given to students. During the review period, students will be asked to take two full AP exams. The midterm exam will be similar to the AP, as well.

Evaluative work

Quizzes are frequent (one per week or more), and there are daily homework sets. The expectation of the course is that there will be 40 minutes of homework (approximately 20 problems). Students that are interested in stretching their knowledge have an optional problem set every week. Tests count together for 70% of the grade, quizzes 10%, and homework and classwork 10% each. Tests gradually become more and more "AP-like" during the year: calculators are allowed only for the first ⅓ of tests, and tests are evenly divided between multiple choice and free response problems. Reflective work with grading is required with every test. Students are asked to justify one answer on every test in paragraph form. Homework assignments include written-answer responses. Free-response questions are evaluated in the second half of the course, with written analyses of scoring applied to free-response samples.

Text:

Calculus: Graphical, Numerical, Algebraic. 3rd edition, by Finney, Demana, Waits, and Kennedy. Pearson Education Publishing, Pearson Prentice-Hall, Boston, Massachusetts. 2007

Other resources:

- www.desmos.com (an excellent online graphing utility)
- Vernier: probes and loggerpro software (www.vernier.com)
- Calclus.org (UCDavis' collection of worked problems, in order of difficulty and organized by topic)
- Khan Academy website
- AP Calculus BC course description (2013)

Course Topical and Chapter Outline **(SC1, SC2)** (approximate number of class days in parentheses)

Chapter 1 – Prerequisites

• BC Calculus students are required to master the prerequisites chapter over the summer prior to beginning the course.

Chapter 2 – Limits and Continuity

• 2-1 Rates of Change and Limits 3
• 2-2 Limits Involving Infinity 2
• 2-3 Continuity 2
• 2-4 Rates of Change and Tangent Lines 2
• Review, Test covering chapter 2 (2)
Sub total days (11)

Chapter 3 – Derivatives (SC2)

• 3-1 Derivative of a Function (4)
• 3-2 Differentiability (1)
• 3-3 Rules for Differentiation (4)
• Review and test: 3-1 through 3-3 (2)

• 3-4 Velocity and Other Rates of Change (2)
• 3-5 Derivatives of Trigonometric Functions (3)
• 3-6 Chain Rule (4)
(Includes lab activity: finding derivatives of nested trig functions using springs and motion detectors)
• Review and test 3-4 to 3-6 (2)

• 3-7 Implicit Differentiation (2)
• 3-8 Derivatives of Inverse Trigonometric
Functions (2)
• 3-9 Derivatives of Exponential and
Logarithmic Functions
• Review and test 3-7 to 3-9 (2)

Subtotal for chapter 3: (20)

Chapter 4 – Derivatives: Applications (SC3)

• 4-1 Extreme Values of Functions (2)
• 4-2 Mean Value Theorem, aka “highway theorem” (3)
• 4-3 Connecting f’ and f” with graph of f (3)
• Review and test 4-1 to 4-3 (2)

• 4-4 Modeling and Optimization (5)
  Includes project: building models of optimization problems
• 4-5 Linearization: Newton’s Method not included (1)
• 4-6 Related Rates (4)
• Review and test 4-4 to 4-6 (2)

Sub-Total Days 22

Chapter 5 – The Definite Integral

• 5-1 Estimating with Finite Sums (3)
• 5-2 Definite Integrals (4)
• 5-3 Definite Integrals and Antiderivatives (4)
• 5-4 Fundamental Theorem of Calculus (4)
• 5-5 Trapezoidal Rule (3)
• Review and Test (2)

Sub-Total Days 20

Chapter 6 – Differential Equations and Mathematical Modeling

• 6-1 Slope Fields and Euler’s Method Separation of variables (4)
• 6-2 Antidifferentiation by Substitution (5)
• 6-4 Exponential Growth and Decay (4)

Review and Test 3

Sub-Total Days 16

Chapter 7 – Definite Integrals: Applications

• 7-1 Integral as Net Change (3)
• 7-2 Areas in the Plane (3)
• 7-3Finding volumes with discs only (3)
• 7-4 Lengths of Curves (3)
• 7-5 Applications for Science and Statistics (3)

Sub-Total Days 15

Chapter 8 – Definite Integrals: Applications

- 8-1 Sequences
- 8-2 L'Hopital's Rule
- 8-3 Relative rates of growth
- 8-4 Improper Intergrals

Sub-Total Days 10

Chapter 9 – Series

- 9-1 Power Series
- 9-2 Taylor Series
- 9-3 Taylor's Theorem
- 9-4 Radius of convergence
- 9-5 Testing convergence at endpoints

Sub-Total Days 10

Chapter 10 – Polar Coordinates

10-1
10-2
10-3

AP Exam Review 30 days, as time permits
Additional Topics (after AP exam) as time permits:

- 6-3 Integration by Parts 3
- 6-5 Logistic Curve 2
- 7-3 Volumes: Shell Method 2
- 7-5 Work Problems 2

Project involving finding volumes using disc/shell method (5)
Days: 14+

TOTAL DAYS 147, without additional topics, 161 with additional topics.